

Engineering a Mathematics Course at the United States Military Academy

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Abstract - Civil & mechanical engineering majors at West Point must complete a five semester mathematics sequence as part of their Accreditation Board for Engineering and Technology accredited academic program. Given the increased interest in multidisciplinary team teaching and the use of technology within their respective academic programs, the Heads of the Departments of Civil & Mechanical Engineering and Mathematical Sciences decided to re-examine the content, structure and teaching of the last course in that five course sequence - MA364 "Engineering Mathematics"- in an attempt to better motivate engineering majors to study key mathematical concepts in detail and to further strengthen inter-departmental ties. This paper describes the two-pronged approach adopted to "engineer" the mathematics course in question. Particular attention is given to the integration of technology across the course and the planned use of a team teaching approach involving engineering and mathematics faculty.

Index Terms - Information technology, team teaching, teaching model, learning styles, feedback, assessment

MA364 "ENGINEERING MATHEMATICS"

Despite the close relationship between engineering and mathematics, prior to the start of the 2003 academic year, sole responsibility for teaching mathematics electives resided with the Department of Mathematical Sciences. In the summer of 2003, I was ordered by my Department Head to help teach a course in "Engineering Mathematics" with the Mathematical Sciences Department at the United States Military Academy. This paper describes the resulting experience.

Course Description

MA364 "Engineering Mathematics" is a three credit hour course taught in 40 lessons over a single semester. As described in the course catalog, the course exists "to provide exposure to additional mathematical techniques and deepen a cadet's understanding of concepts in mathematics to support continued study in science and engineering". Emphasis is placed upon using mathematics to gain insight into natural and man-made phenomena that give rise to problems in differential equations and vector calculus. Calculus topics focus on three-

dimensional space curves, vector fields and operations, divergence and curl, line and surface integrals. Analytic and numerical solutions to differential equations and systems of differential equations are found using a variety of techniques, including the use of Laplace Transforms. Linear algebra topics include solutions to homogeneous and non-homogeneous systems of equations. An introduction to classical partial differential equations is included in the course as well.

Initial Course Assessment

Having no background with this particular course, I arranged a few meetings with the current course director, Dr. Brian Winkel, in order to get a feel for the course structure and content. There is a very comprehensive end-of-course feedback system here at West Point so I also wanted to get the results from several recent semesters in order to gain a sense of how the course was perceived by the target audience. Dr. Winkel was extremely helpful and made me feel welcome from the first day we met. This sense of "team" was important as we would spend many hours that semester talking about pedagogy. By the end of the semester my initial misgivings about teaching the course would prove unfounded. I would learn more about teaching and course design while helping with this course than I did in three years of teaching engineering mechanics courses. After our first meeting though those realizations had yet to materialize. Dr. Winkel asked me to prepare a draft syllabus as a basis for discussion. I left his office with the feedback results and the URL to the course website with last semester's syllabus, homework assignments, exams and policy letters.

Course Content

My review of the course website revealed that the mathematical concepts taught in the most recent version of the engineering mathematics course fell into a few key topic areas: 2nd order ordinary, linear differential equations, systems of ordinary, linear differential equations, integration and vector operations. Within each major area, several different mathematical concepts were studied in more detail. In the case of 2nd order differential equations, harmonic, periodic and impulsive forms of loading functions were applied so that homogeneous and particular solutions could be evaluated and compared. LaPlace transforms were also introduced to explore

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frequency domain solution techniques. Systems of differential equations permitted the study of eigenvectors and eigenvalues.

In addition to differential calculus topics, integral calculus concepts seen in earlier courses were reinforced and included integration in two- and three- dimensions, vector dot products and vector cross products.

Graded Requirements

Cadets were graded daily on work completed out-of-class. There were three in-class exams and a term-end exam. The course also contained a group project where two or three cadets would select an engineering topic and examine the mathematics governing that topic in detail. A formal report and a presentation were required.

Cadet Feedback

At the United States Military Academy, the cadets provide course assessment feedback via an anonymous, web-based system. Feedback results from several recent semesters were reviewed to get a sense of student satisfaction with the course. In general, the data indicated the cadets were not happy with the pace or structure of the course. They felt the number of graded assignments was excessive and left little time for reflection. The cadets did not like using the mathematical software program *Mathematica* [1] which had been recently adopted for use by the Mathematical Sciences Department. Perhaps the most troubling observation was that the cadets did not see the relevance of the course to their engineering studies. I concluded that the target audience consisted of a fairly homogenous group in terms of academic major and academic background who felt they were being asked to do too much in too short a time while using a software program that they did not like. With this general observation in mind, I decided to look for a way to structure the course to increase cadet motivation to study.

Target Audience & Course Scheduling

Civil and mechanical engineers dominate the student population in the course with most civil engineers taking the course in the fall semester junior year. Mechanical engineers generally take the course in the spring of their sophomore year as do electrical engineers. A total of 41 cadets completed MA364 "Engineering Mathematics" in the Fall semester of 2003. Of those, 36 were civil engineering majors, four were electrical engineering majors and one was a mechanical engineering major. Forty of the cadets were 1st semester juniors. There was one 1st semester sophomore with a strong mathematics background in the course.

THE C&ME TEACHING MODEL

The Department of Civil & Mechanical Engineering prides itself on the model it uses to design courses and present instruction. The "C&ME Model" is backed by educational research and currently serves as the pedagogical framework in the American Society of Civil Engineer's Excellence in Civil Engineering Education (ExCEED) workshop hosted every year

at several locations across the United States. The focus of the teaching model is on "effective teaching" which has its roots in Lowman's 2-dimensional teaching model. [2] In his model, Lowman defines exemplary teaching as a function of intellectual excitement and interpersonal rapport. Intellectual excitement refers principally to teacher performance. Specific metrics include technical expertise, organization, ability to communicate clearly, an engaging presentation style and enthusiasm. [3] The justification for all changes made to MA364 "Engineering Mathematics" can be traced to this teaching model.

Course Organization

Given the limited time available prior to the start of the fall semester, I decided to maintain the mathematical concepts taught in previous versions of the course as little course assessment feedback from cadets, mathematics faculty or engineering faculty existed to justify doing otherwise. There seemed to be no need to drop or add topics. What was needed was a conceptual framework for teaching them.

The C&ME Model calls for a structured organization based on learning objectives identified for each lesson which are appropriate to the matter being studied and apply one or more learning styles. [4] Course structure though defines the resulting course organization. To mitigate much of the negative cadet feedback regarding previous versions of the course, and to take advantage of the presence of professors with engineering and mathematical expertise, I made two decisions which would determine the eventual course structure and organization. The first decision was that the revised course would place all mathematical concepts in an engineering context. Research exists to support the conclusion that a person's ability to solve problems depends on having access to experiences organized in a contextual framework. [5] The second decision was to maximize the use of information technology throughout the course. I asked Dr. Winkel if I could develop the course around *MathCad*, a software program used in many civil engineering courses here at West Point. As the Mathematical Sciences department had already adopted *Mathematica* and used it in its courses, my request was denied, so I set about learning how to use *Mathematica*. The end result of these two decisions is seen in the schematic below (1) which I recommended govern the structure of the course.

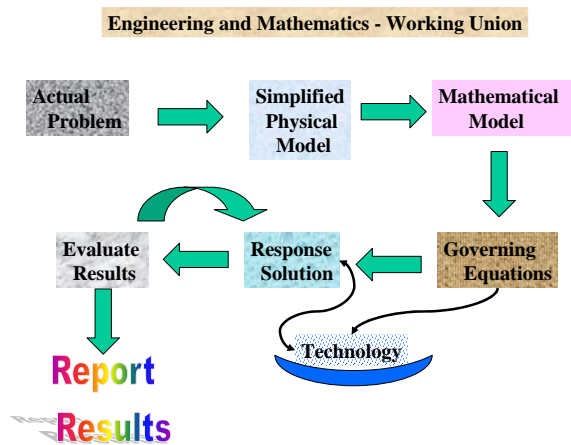


FIGURE 1
ENGINEERING & MATHEMATICS

Presentation

Using this approach, I placed each mathematical concept in an engineering context. Each block of lessons devoted to a mathematical concept would augment the development of governing equations based on fundamental principles using *Mathematica* to solve those same equations. Cadets would gain an appreciation of the physics and engineering principles which underpinned these equations while also gaining experience with a powerful computational and tremendous visualization tool, in this case, *Mathematica*. My goal was to demystify the software program and force the cadets to see *Mathematica* only as a tool to be used in engineering analysis and design.

2nd order, ordinary, linear differential equations were examined using vibrations of mechanical systems subjected to various loading forms. Simple LRC circuits offered a means of showing cadets how the same basic principles underpinned the analysis of both mechanical and electrical systems.

1st order, ordinary, linear differential equations were introduced to allow the development of two dimensional heat analysis using structural members.

Partial differential equations were studied using longitudinal and shear waves in soil as well as heat diffusion.

Systems of ordinary, linear differential equations were examined using vibrations of two degree-of-freedom mechanical systems.

Finally, a few lessons were devoted solely to integration and vector operations such as flux, dot product and cross product.

A team teaching approach provided an opportunity to maximize each instructor's subject matter expertise for the cadet's benefit. Actual instruction occurred in three distinct phases. In the first third of the course which was devoted solely to 2nd order, linear, homogeneous differential equations with constant coefficients, I taught my section during the first class hour while Dr. Winkel sat through the presentation. Dr. Winkel felt doing this would provide him with a better

understanding of the engineering applications being used to provide context for the mathematical concepts in that lesson. During the middle third of the course, Dr. Winkel taught the integration, vector operation and classical heat equation concepts to all three sections. In the remaining third of the course, I taught the material concerning systems of differential equations to all three sections. At times, we would each attend the other's classes. Both instructors were always in attendance during graded events and computer lab sessions. This allowed one instructor to focus on teaching while the other assisted as needed.

Use of Technology

Technology was embedded throughout the course in several different forms. During the 1st third of the course, we made extensive use of a computer lab just 2 rooms down the hall from our classroom. I assembled the lesson notes so that every 2nd or 3rd lesson, we would take the cadets to the computer lab where we would expose them to simple problems designed to increase their comfort level with *Mathematica* and allow them to start solving the governing equations developed in class.

In the 2nd third of the course, we stopped using the computer lab and stayed in our classroom which was equipped with a personal computer and an Epson® projection system. In an attempt to build upon theory just developed in class, we would assign one cadet to act as a class typist at the computer keyboard while all of the cadets responded to instructor and cadet questions/stimuli and developed solutions to problems. With *Mathematica*'s powerful graphical tools, we were able to readily explore answers to cadet "what if..." and "what happens when..." questions. All files were saved from each class and posted to a comprehensive course website so that the cadets could download them to their personal computers later. This approach proved so successful and popular with the cadets that we never returned to the computer lab for the remainder of the course except for exams. Current versions of the course are taught in a computer rich environment in which all cadets have their own wireless laptops and find it rewarding to build their own files during class and problem solving labs.

Extensive use of a course website proved invaluable. All solutions for graded assignments were prepared using *Mathematica* and placed on the course website. Professor biographies, contact information and a resource rich course syllabus supplemented the solution library. All cadets had access to the notes from all three sections.

Graded Assignments

In an attempt to create time for deeper study and reflection, I recommended that we eliminate the daily graded homework and replace it with a series of eight problem sets supplemented by two exams and two engineering analysis projects (one individual and one group).

The cadets were given three weeks to complete the first engineering analysis project. The cadets had seven weeks to complete the second project, which they had to choose and design themselves. Each project required one or more in-progress reviews to keep the cadets on target and help them

refine their project focus. Formal reports and a final presentation were required for the group project. Both my colleague and I realized that “delayed” starts on work at home was not unique to our discipline but we felt that a series of intermediate progress review efforts by the cadets would result in better final products and less frustration on the part of the cadets. .

The cadets were required to complete all problem sets using Mathematica. Both exams were given in a computer lab and cadets were given free access to their own computers in their rooms as well as the course website and internet in general. Exam equations were phrased such that cadets merely had to transcribe answers or sketch graphs based on calculations they performed in response to a given question.

Course Assessment

Periodically during the course, I sought feedback from my section. In each case, I used a different format to gather their feedback anonymously. Based on this feedback, I would recommend slight adjustments to the course. In several instances, the feedback supported the initial changes made to the course at the start of the semester.

One tool I used as a measure of cadet “time on task” was the Time Survey. At the start of each class, each cadet would write down the number of minutes spent preparing for class since the previous lesson. Even without labels, one can readily identify lessons when the 10 out-of-class graded assignments were due. By the end of the semester, the cumulative trend line depicts an average class preparation time of 74 minutes per cadet.

directors and individual instructors to ask specific questions of interest. Because of the initial decision to restructure the course to increase cadet motivation to study by placing all mathematical concepts in an engineering context and as a result increase their abilities to “think” and solve problems of engineering interest, I suggested that we use the same questions asked by the Civil & Mechanical engineering department in the general portion of their end-of-course survey. The results for questions common to the Mathematical Sciences and Civil & Mechanical Engineering Departments (4) and the questions borrowed from the Civil & Mechanical Engineering Department (5) are shown.

The following scale applies to Figures (4) and (5).

- Strongly Agree..... 5
- Agree..... 4
- Neutral..... 3
- Disagree..... 2
- Strongly Disagree..... 1

The instructor questions (4) help assess student motivation to study by asking if the cadets felt their instructor was a good role model worth emulating who basically came to class prepared. Preparedness here, in addition to supporting a good learning environment, implies professional courtesy and respect. The course questions (5) help assess intellectual growth in terms of intellectual stimulation, self-motivation and self-perceptions regarding problem solving ability.

DISCUSSION

Although the sample population is somewhat small, I believe a few general observations about the effectiveness of the team teaching and IT integration approach can be made based on the cadet feedback results. A measure of effectiveness can be made by assessing changes in a cadet’s intellectual development. Perry’s Scheme of Intellectual and Ethical Development [6], offers a framework for performing just such an assessment in order to identify the relationship between learners and knowledge.

At this stage in their academic program, most cadets fall into Position 2 (Dualism) with a small number in Position 3 (Multiplicity). In the dualism stage, learners are very “teacher oriented”. By that I mean that the students feel the teacher is the expert and the font of all knowledge. The student’s role is simply to sit, watch, listen and regurgitate answers when required. In this stage, the cadets see my role as teacher as one of simply “passing the poop”.

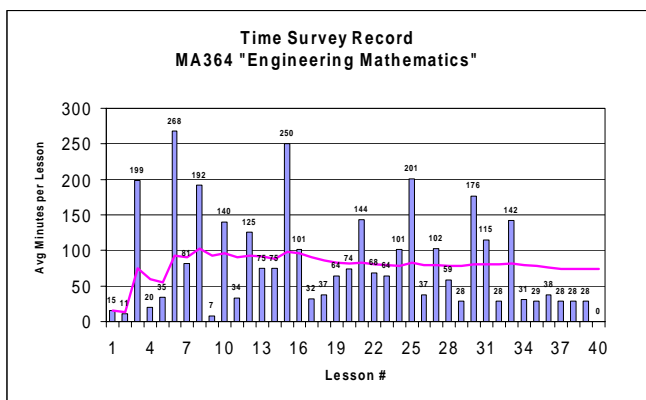


FIGURE 2
TIME SURVEY RECORD

Using the same end-of-course feedback system described earlier, Dr. Winkel and I were able to gather feedback concerning the changes made to MA364, “Engineering Mathematics”. The feedback system is set up to allow course

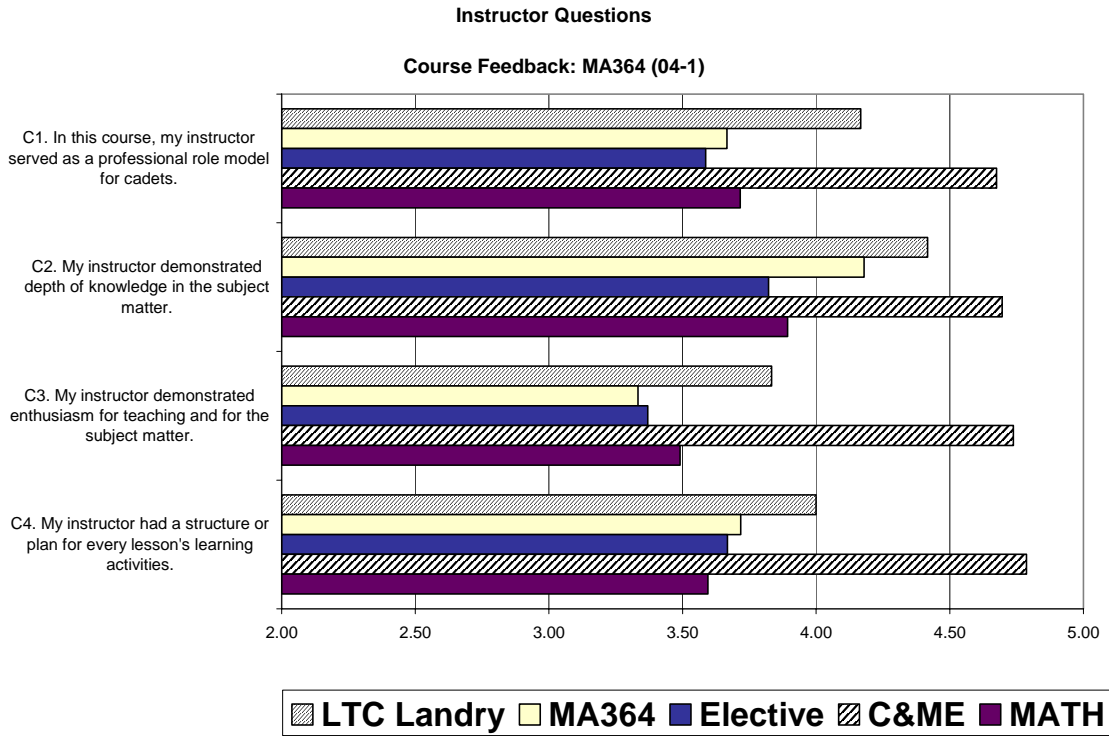


FIGURE 4
CADET END-OF-COURSE FEEDBACK: MATH & C&ME

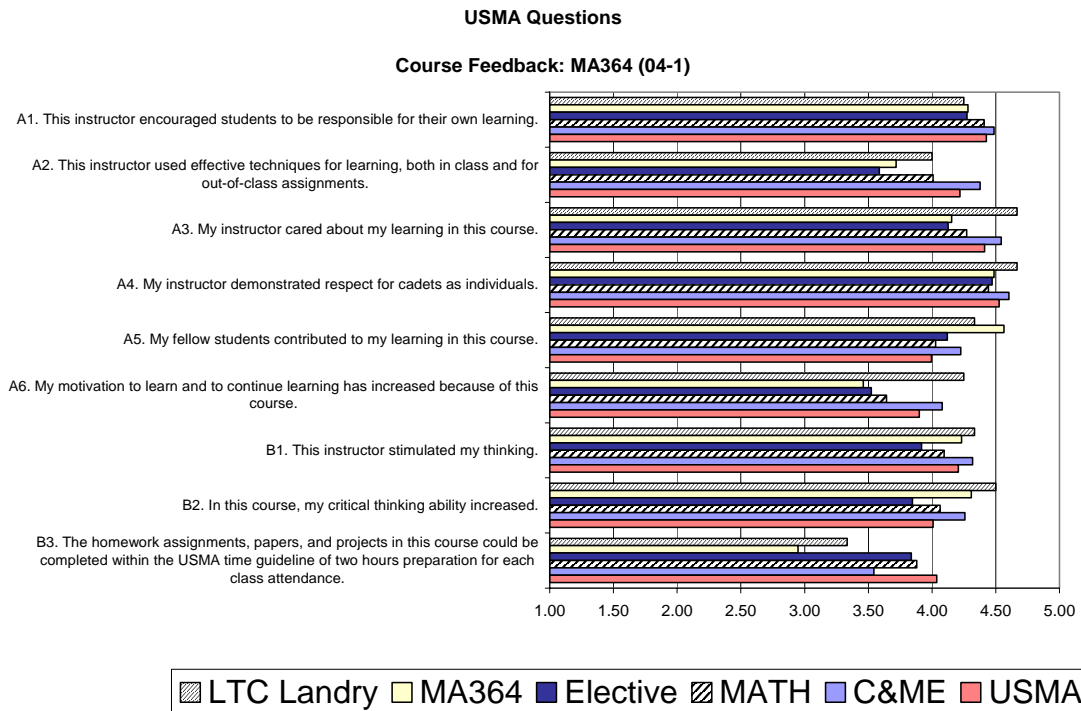


FIGURE 5
CADET END-OF-COURSE FEEDBACK: C&ME

Of the 41 cadets who finished the semester, I believe roughly 10% never left this stage despite our best efforts. These efforts included activities with instructions to “explore,” “compare,” “explain,” and “communicate,” all designed to move students beyond the black and white answer state of Dualism. Indeed, almost all of the inquiry required of the cadets, as defined in the lesson objectives and problem statements embedded in each lesson and graded homework assignment was well beyond the simple “solve” and “find” sort of question, principally because the technology (*Mathematica*) could do that in a keystroke but also because we were trying to develop engineers who could solve problems yet unseen. Thus the emphasis on modeling, on examining changes in parameters, on open-ended questions for example was enabled by technology and we could see them grow as they stopped asking, “What do you want?” and began addressing the broader issues outlined in the problems they were assigned.

In the Multiplicity stage, students understand that while much of what they will learn is already well-defined there are some areas which offer room for exploration. For cadets in this stage, my role as their teacher is to help the cadets learn the skills required to take what they already know, extrapolate when required and work their way through new and challenging problems which have more than one viable solution. I think of students in this stage as “reluctant learners”. If the teacher does not provide interesting problems and motivational solution methods, the students will resist further growth toward Stage 4 (Relativism) and perhaps even trend backwards toward stage 2 (Dualism). Of the 41 cadets who finished the semester, the answers to questions A.1, A.2, A.6 B.1 and B.2 support the conclusion that a majority of cadets reached stage 3 (Relativism) because of the analytical process used to structure the course as seen in (1) and the placement of each mathematical concept in an engineering context.

CONCLUSIONS

Team teaching and information technology are useful pedagogical tools but they are not panaceas. As tools they require thoughtful application at the right times in the right places. Feedback from our target population indicates that the structured application of information technology and a team

teaching approach proved synergistic in MA364 “Engineering Mathematics”.

I believe having the cadets see two professors in the same course, sometimes in the same class, was beneficial. Every class was well structured so we did not offer conflicting views on the material. Dr Winkel became the subject matter expert on “mathematics” while I became the expert on “engineering application”. I believe many cadets felt comfortable going to see either of us for additional instruction on most topics. Indeed, a major component of the coursework for the cadets was a project in an area of their choice and they sought mentoring from both of us regularly. Evidence of the cadet comfort level with this team teaching approach can be seen in one cadet’s response to the query, “Who is your MA364 teacher – LTC Landry or Dr. Winkel?” from my department head. The cadet’s answer was simply, “Sir, I think I have them both”!

ACKNOWLEDGMENT

I am a better engineer for having taught this course with Dr. Brian Winkel. I find that I miss our talks. I am also thankful to Colonel Kip Nygren, Head, Department of Civil & Mechanical Engineering and Colonel Gary Krahn, Head, Department of Mathematical Sciences, for this fusion of engineering and mathematics faculty. It was a challenging and a rewarding experience for me while serving as a fruitful one, I believe, for the cadets.

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